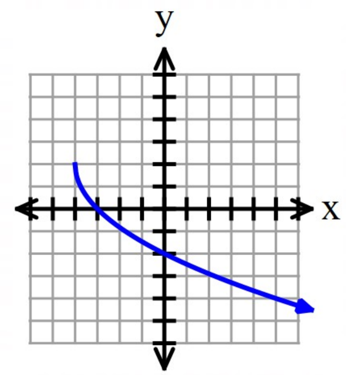
Math 126 **S1 Practice Final** **Name______________________________**

Scientific calculator is allowed.

$\frac{n}{2}(a_1 + a_n)$	$a_1 r^{n-1}$	$\frac{a_1}{1-r}$
$a_1 + d(n-1)$ or $dn + a_0$	$\frac{a_1(1-r^n)}{1-r}$	$\frac{P\left[\left(1+\frac{r}{n}\right)^{nt}-1\right]}{\frac{r}{n}}$

1) Given that $f(x)$ as graphed to the right, write an equation for $f(x)$.



The curve of this function indicates that it is a square root function.

The standard form of a square root function is:

$y = a\sqrt{x-h} + k$, where (h, k) is the vertex.

In the graph shown, $(h, k) = (-4, 2)$, so the equation becomes:

$y = a\sqrt{x+4} + 2$

Now, select a point elsewhere on the curve, say $(0, -2)$. Substitute this point into the equation and solve for a.

$-2 = a\sqrt{0+4} + 2$

$-4 = 2a$

$a = -2$

Therefore, the potential equation becomes:

$\boldsymbol{y = -2\sqrt{x+4} + 2}$

To be sure we picked the right form for the equation $\left(a\sqrt{x-h} + k\right)$, select a point elsewhere on the curve, say $(5, -4)$. Substitute this point into the equation and see if the result is true. If it is, we are done; if not, we must try another form for the equation.

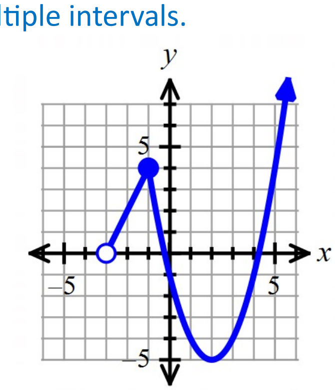
$-4 = -2\sqrt{5+4} + 2$

$-4 = -2(3) + 2$

$-4 = -6 + 2 = -4$ ✓ We are done.

2) State the following intervals from the provided graph (use interval notation):

For increasing and decreasing, use open intervals containing x-values.
For more than one interval, use set notation to indicate a union of multiple intervals.



Increasing: $(-3, -1) \cup (2, \infty)$

Decreasing: $(-1, 2)$

Domain: $(-3, \infty)$ are the x-values of the curve.

Range: $[-5, \infty)$ are the y-values of the curve.

3) Evaluate the function at the given value of the independent variable and simplify.
$f(x) = 4x^2 + 2x + 6; f(x-1)$

$f(x) = 4x^2 + 2x + 6$

$$\begin{aligned} f(x-1) &= 4(x-1)^2 + 2(x-1) + 6 \\ &= 4(x^2 - 2x + 1) + 2(x-1) + 6 \\ &= 4x^2 - 8x + 4 + 2x - 2 + 6 \\ &= \mathbf{4x^2 - 6x + 8} \end{aligned}$$

4) Find $g(2)$.

$$g(x)\begin{cases} \frac{x^2-6}{x+5} & if\ x \neq -5 \\ x+6 & if\ x = -5 \end{cases}$$

To find the value of $g(2)$, we must look at the conditions given in the definition of the piecewise function, i.e., the "if" parts of the definition. In this problem, since $x \neq -5$, we must use the top of the two pieces given. So,

$$g(x) = \frac{x^2-6}{x+5} \quad \Rightarrow \quad g(2) = \frac{2^2-6}{2+5} = \frac{\mathbf{-2}}{\mathbf{7}}$$

5) Given $f(x) = 2x^2 - 3x + 5$, then find $\frac{f(x+h)-f(x)}{h}$.

$$\begin{aligned} \frac{f(x+h)-f(x)}{h} &= \frac{[2(x+h)^2 - 3(x+h) + 5] - [2x^2 - 3x + 5]}{h} \\ &= \frac{[2(x^2 + 2hx + h^2) - 3(x+h) + 5] - [2x^2 - 3x + 5]}{h} \\ &= \frac{[2x^2 + 4hx + 2h^2 - 3x - 3h + 5] - [2x^2 - 3x + 5]}{h} \\ &= \frac{[4hx + 2h^2 - 3h]}{h} = \mathbf{4x + 2h - 3} \end{aligned}$$

For 6 – 7: Find the inverse of the one-to-one function.

6) $f(x) = \frac{2x+5}{7}$

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = \frac{2y+5}{7} \quad \Rightarrow \quad 7x = 2y + 5 \quad \Rightarrow \quad 7x - 5 = 2y \quad \Rightarrow \quad \frac{7x-5}{2} = y$$

So,

$$\mathbf{f^{-1}(x) = \frac{7x-5}{2}}$$

7) $f(x) = \sqrt{x+8}$

To find an inverse function, switch the x and y in the original function and solve for y.

$$x = \sqrt{y+8} \quad \Rightarrow \quad x^2 = y + 8 \quad \Rightarrow \quad x^2 - 8 = y$$

So,

$\boldsymbol{f^{-1}(x) = x^2 - 8}$**,** with the restriction that $x \geq 0$ because $y \geq 0$ in the original function.

8) Divide the following using long division, $(6x^2 + 17x - 45) \div (3x - 5)$.

$$\begin{array}{lr} & 2x + \ \ 9 \\ 3x - 5 \ \big) & 6x^2 + 17x - 45 \\ CS & -6x^2 + 10x \\ \hline & 27x - 45 \\ CS & -27x + 45 \\ \hline & 0 \end{array}$$

Long division of polynomials is just like long division of numbers. The lone difference is indicated by the notations ***CS***, which indicates that the signs of the terms in the row have been changed to allow addition of rows instead of subtraction.

The result of the division is shown on the top line: $\boldsymbol{2x + 9}$

For #9 – 10: Solve the polynomial equation for all zeros. Include imaginary solutions, if applicable.

9) $x^3 + 2x^2 - 5x - 6 = 0$

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm \frac{1,2,3,6}{1} = \pm 1, \pm 2, \pm 3, \pm 6$. Let's consider them:

- $+1$ will not work because the sum of the coefficients is not zero.
- -1 will work because the sum of the coefficients of the odd degree terms $(1 - 5 = -4)$ is equal to the sum of the coefficients of the even degree terms $(2 - 6 = -4)$.

Let's use synthetic division with -1 as a root (or "zero").

$$\begin{array}{r|rrrr} -1 & 1 & 2 & -5 & -6 \\ & & -1 & -1 & 6 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

The remaining polynomial is $x^2 + x - 6 = (x+3)(x-2) \quad \Rightarrow \quad x = \{-3, 2\}$

So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-3, -1, 2\}}$

10) $x^3 + 7x^2 + 19x + 13 = 0$

Possible rational roots are of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the lead coefficient. So, possible rational roots are: $\pm\frac{1,13}{1} = \pm 1, \pm 13$. Let's consider these:

- Descartes Rule of Signs tells us that there are no positive real roots because there are no sign changes in the polynomial. That leaves us with -1 and -13 as possible roots.
- -1 will work because the sum of the coefficients of the odd degree terms $(1 + 19 = 20)$ is equal to the sum of the coefficients of the even degree terms $(7 + 13 = 20)$.

Let's use synthetic division with -1 as a root (or "zero").

$$\begin{array}{r|rrrr} -1 & 1 & 7 & 19 & 13 \\ & & -1 & -6 & -13 \\ \hline & 1 & 6 & 13 & \boxed{0} \end{array}$$

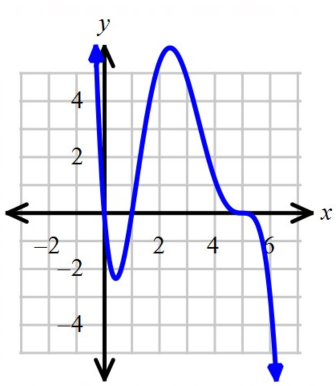
The remaining polynomial is $x^2 + 6x + 13$. To get the remaining roots, let's use the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-6 \pm \sqrt{36 - 4(1)(13)}}{2(1)} = \frac{-6 \pm \sqrt{-16}}{2} = \frac{-6 \pm 4i}{2} = -3 \pm 2i$$

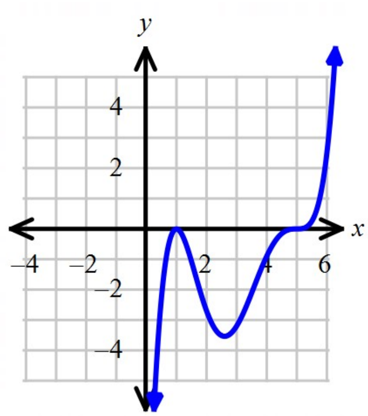
So, the **complete solution** of the polynomial is the set of zeros: $\boldsymbol{x = \{-1, -3 \pm 2i\}}$

11) **Multiple Choice.** Which could be the graph of $p(x) = \frac{1}{10}x(x-1)(x-5)^3$?

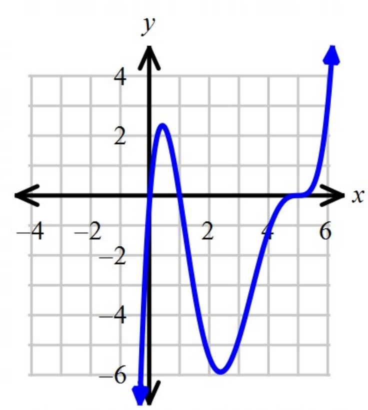
A)



B)



C)



Where a function has a zero with an odd multiplicity, the graph flows through the graph at that x-value.

Where a function has a zero with an even multiplicity, the graph bounces off the graph at that x-value.

$p(x)$ has odd multiplicities at all three of its zeros, $x = 0, 1, 5$.

The graphs that flow through the graph at $x = 0, 1, 5$ are **A** and **C**. However, our function has a positive lead coefficient, like **C** but not like **A.** Therefore, the correct graph is **C.**

12) Given the rational function $h(x)$, which statements are **FALSE**? $h(x) = \frac{x^2-9}{x^2-2x-3}$ **Select all that apply.**

Let's look at the function in question:

$$h(x) = \frac{x^2-9}{x^2-2x-3} = \frac{(x-3)(x+3)}{(x-3)(x+1)} = \frac{(x+3)}{(x+1)}$$

What can we see from this?

i. The factor $(x-3)$ disappeared from the denominator after simplification. Therefore, there is a hole in the graph at $x = 3$.

ii. The factor $(x+1)$ remained in the denominator after simplification. Therefore, there is a vertical asymptote in the graph at $x = -1$.

iii. As x gets larger and larger, $h(x) = \frac{(x+3)}{(x+1)}$ gets closer to 1. Therefore, there is a horizontal asymptote at $y = 1$.

Consider each answer:

A) $h(x)$ has a vertical asymptote at $x = 3$. **FALSE** because of i above.

B) $h(x)$ has a horizontal asymptote at $y = 1$. TRUE because of iii above.

C) $h(x)$ does not have a slant asymptote. TRUE because the numerator and denominator of $h(x)$ have the same degree.

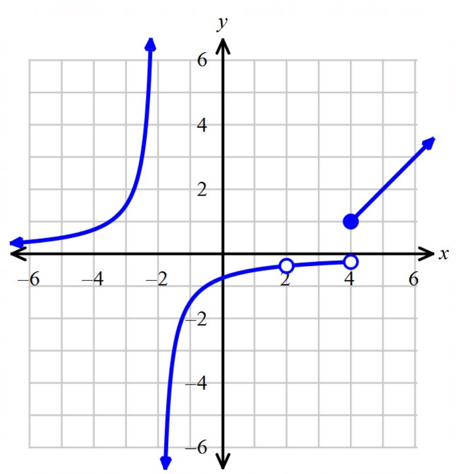
D) $h(x)$ has an x-intercept at (-3, 0). TRUE because the numerator is zero at $x = 3$ and there is neither a hole nor a vertical asymptote at $x = 3$.

E) $h(x)$ has a y-intercept at $(0,9)$. **FALSE** because when $x = 0$, $y = 3$.

F) $h(x)$ has a hole at $(3,0)$. **FALSE** because there is a hole at $x = 3$, but $y \neq 0$. y is determined from the simplified form of $h(x)$. The hole has $y = \frac{(x+3)}{(x+1)} = \frac{(3+3)}{(3+1)} = \frac{6}{4} = \frac{3}{2}$.

Solution: A, E, F

13) **Multiple Select.** Given the graph of $f(x)$ as shown below, which statements below correctly describe the discontinuities? **Select all that apply.**



A) non-removable at $x = -2$ (jump discontinuity). FALSE because the graph has a vertical asymptote at $x = -2$.

B) non-removable at $x = -2$ (vertical asymptote; infinite) **TRUE** because the graph $\to \pm\infty$ at $x = -2$.

C) removable at $x = 2$ (hole) **TRUE** because of the hole in the graph at $x = 2$.

D) non-removable at $x = 2$ (hole) FALSE because holes are removable, not non-removable.

E) non-removable at $x = 4$ (jump discontinuity) **TRUE** because the graph "jumps" at $x = 4$. Jumps are non-removable.

F) removable at $x = 4$ (jump discontinuity) FALSE because jumps are non-removable, not removable.

Solution: B, C, E

14) Find equations of all asymptotes, if any, of the graph of the rational function $f(x) = \frac{x^2-3x+2}{x+5}$.

Horizontal asymptote: **none** since the degree of the numerator is greater than the degree of the denominator.

Slant asymptote: exists whenever the numerator is one degree higher than the denominator. That is the case with this function, so a slant asymptote exists. To obtain the slant asymptote, divide the numerator by the denominator and disregard any fractional component of the answer.

Use synthetic division to obtain the slant asymptote. Recall that the divisor term is the root of the denominator of the function, i.e., -5. Then,

$$\begin{array}{r|rrr} -5 & 1 & -3 & 2 \\ & & -5 & 40 \\ \hline & 1 & -8 & \boxed{42} \end{array}$$

The result of the division, excluding the fractional component, provides the slant asymptote:

$$y = x - 8$$

Vertical asymptote: exists whenever the denominator of the simplified fraction of a rational function is zero.

$$\frac{x^2-3x+2}{x+5} = \frac{(x-2)(x-1)}{x+5}$$

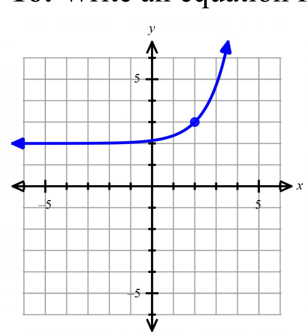
Since this expression cannot be simplified, a vertical asymptote exists where $x + 5 = 0$, i.e. at:

$$x = -5$$

Final answer: Asymptotes exist at: $y = x - 8$ and $x = -5$

For 15 – 16: Write an equation for the following graphs. Use the given equations under the graphs.

15)

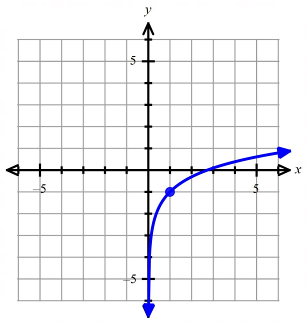


$y = 3^{x-\_\_} + \_\_\_\_$

For the parent of an exponential graph, the anchor point is $(0, 1)$. This graph has an anchor at $(2, 3)$, which requires translation of the parent graph two units right $(x - 2)$ and two units up $(+2)$. The resulting function, then, is:

$$\boldsymbol{y = 3^{x-2} + 2}$$

16)



$y = \ln(x - \_\_\_) + \_\_\_$

For the parent of a logarithmic graph, the anchor point is $(1, 0)$. This graph has an anchor at $(1, -1)$, which requires translation of the parent graph one unit down (-1). The resulting function, then, is:

$$\boldsymbol{y = (\ln x) - 1}$$

17) Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ and $A = Pe^{rt}$ to solve. Find the accumulated value of an investment of \$6000 at 8% compounded semiannually for 8 years. Then find the accumulated value of the same investment but compounded continuously.

Compounded semiannually: $P = \$6000$, $r = 8\% = .08$, $n = 2$ (semiannual), $t = 8$ years. Then,

$$\boldsymbol{A} = \$6000\left(1 + \frac{.08}{2}\right)^{2\cdot 8} = \$6000 \cdot 1.04^{16} = \boldsymbol{\$11,237.89}$$

Compounded continuously: $P = \$6000$, $r = 8\% = .08$, $t = 8$ years. Then,

$$\boldsymbol{A} = \$6000 \cdot e^{8(0.08)} = \$6000 \cdot e^{0.64} = \boldsymbol{\$11,378.89}$$

For #18 – 19: Write the equation in the equivalent exponential or logarithmic form.

For these two problems, we will convert using the first-last-middle trick. That is, the parameters of the converted expression, use the first, last, and middle parameters, in that order, of the unconverted expression.

18) $\log_b 64 = 2$

In the log expression, $\log_b 64 = 2$, **first** is "b", **last** is "2" and **middle** is "64." We put these in an exponential expression, from left to right, to get: $\boldsymbol{b^2 = 64}$.

19) $13^x = 169$

In the exponential expression, $13^x = 169$, **first** is "13", **last** is "169" and **middle** is "x." We put these in a logarithmic expression, from left to right, to get: $\boldsymbol{\log_{13} 169 = x}$.

For #20 – 25: Evaluate the expression without a calculator.

20) $\log_{64} 4$

In the log expression, $log_{64} 4 = x$, **first** is "64", **last** is "x" and **middle** is "4." We put these in an exponential expression, from left to right, to get: $64^x = 4$, then solve:

$log_{64} 4 = x$ converts to: $64^x = 4$ $\longrightarrow$ $\boldsymbol{x = \frac{1}{3}}$

Note: to solve the converted equation, we need to know that $\sqrt[3]{64} = 64^{1/3} = 4$.

21) $\log_5 \frac{1}{\sqrt{5}}$

In the log expression, $log_5 \frac{1}{\sqrt{5}} = x$, we can first simplify the expression to $log_5 \left(5^{-1/2}\right) = x$. Then, **first** is "5", **last** is "x" and **middle** is "$5^{-1/2}$." We put these in an exponential expression, from left to right, to get: $5^x = 5^{-1/2}$, then solve:

$log_5 \frac{1}{\sqrt{5}} = x$ converts to: $5^x = 5^{-1/2}$ $\longrightarrow$ $\boldsymbol{x = -\frac{1}{2}}$

22) $\log_7 7^{18}$

In this log expression, the base of the log is the same as the base of the exponential term. The log and the base of the exponential term cancel and we are left with the exponent:

$\boldsymbol{\log_7 7^{18} = 18}$

23) $\log \left(\frac{1}{1000}\right)$

In the log expression, $log_{10} \left(\frac{1}{1000}\right) = x$, we can first simplify the expression to $log_{10}(10^{-3}) = x$. Then, **first** is "10", **last** is "x" and **middle** is "10^{-3}." We put these in an exponential expression, from left to right, to get: $10^x = 10^{-3}$, then solve:

$log_{10} \left(\frac{1}{1000}\right) = x$ converts to: $10^x = 10^{-3}$ $\longrightarrow$ $\boldsymbol{x = -3}$

24) $\ln e$

In this log expression, the base of the log is the same as the base of the exponent. The log and the base of the exponential term cancel and we are left with the exponent:

$\ln e = \log_e e^1 = \mathbf{1}$

25) $27^{2\log_3 4x}$

$$27^{2\log_3 4x} = (3^3)^{2\log_3 4x} = (3^6)^{\log_3 4x} = (3^{\log_3 4x})^6 = (4x)^6 = 4^6 x^6 = \boldsymbol{4096x^6}$$

For #26 – 31: Expand or condense the expression. Simplify if possible.

26) $\log_2(8x)$

$\log_2 8x = \log_2 8 + \log_2 x = \mathbf{3 + \log_2 x}$

27) $\log_5\left(\frac{125}{x}\right)$

$\log_5\left(\frac{125}{x}\right) = \log_5 125 - \log_5 x = \mathbf{3 - \log_5 x}$

28) $3\log_x 4 + \log_x 2$

$3\log_x 4 + \log_x 2 = \log_x (2^2)^3 + \log_x 2 = \log_x 2^6 + \log_x 2 = \log_x (2^6 \cdot 2) = \mathbf{\log_x 128}$

29) $5\ln x - \frac{1}{3}\ln y$

$5\ln x - \frac{1}{3}\ln y = \ln x^5 - \ln \sqrt[3]{y} = \mathbf{\ln \frac{x^5}{\sqrt[3]{y}}}$

30) $\log_2\left(\frac{x^2}{y^7}\right)$

$\log_2\left(\frac{x^2}{y^7}\right) = \log_2(x^2) - \log_2(y^7) = \mathbf{2\log_2 x - 7\log_2 y}$

31) $\log_5\left(\frac{\sqrt{x}}{25}\right)$

$\log_5\left(\frac{\sqrt{x}}{25}\right) = \log_5(\sqrt{x}) - \log_5(25) = \mathbf{\frac{1}{2}\log_5 x - 2}$

For 32 – 33: Solve the exponential equation. Exact answers only (no decimals).

32) $5^{x+7} = 3$

$5^{x+7} = 3 \quad \Rightarrow \quad (x+7)\ln 5 = \ln 3 \quad \Rightarrow \quad (x+7) = \frac{\ln 3}{\ln 5} \quad \Rightarrow \quad \mathbf{x = \frac{\ln 3}{\ln 5} - 7}$

33) $e^{x+4} = 2$

$e^{x+4} = 2 \quad \Rightarrow \quad (x+4)\ln e = \ln 2 \quad \Rightarrow \quad (x+4) = \ln 2 \quad \Rightarrow \quad \mathbf{x = \ln 2 - 4}$

For #34 – 35: Solve the logarithmic equation, and reject any extraneous solutions.

34) $\log_6 x + \log_6(x - 35) = 2$

Starting equation:	$\log_6 x + \log_6(x - 35) = 2$
Combine log terms:	$\log_6[x \cdot (x - 35)] = 2$
Take 6 to the power of both sides:	$6^{\log_6[x\cdot(x-35)]} = 6^2$
Simplify:	$x \cdot (x - 35) = 36$
Distribute x:	$x^2 - 35x = 36$
Subtract 36:	$x^2 - 35x - 36 = 0$
Factor:	$(x - 36)(x + 1) = 0$
Determine solutions for x:	$x = \{36, -1\}$
Test the solutions of x:	$x = 36$: $\log_6 36 + \log_6(36 - 35) = 2$ ✓
	$x = -1$: $\underbrace{\log_6(-1)} + \underbrace{\log_6(-1 - 35)} = 2$ **X**
Final solution: **$x = 36$**	These terms are both Invalid because negative numbers are not in the domain of the log function.

Note: To test the solutions you derive, use the original equation or a simplified form of the original equation.

35) $\log\,(x + 4) = \log\,(5x - 5)$

Starting equation:	$\log_{10}(x + 4) = \log_{10}(5x - 5)$
Take 10 to the power of both sides:	$10^{\log_{10}(x+4)} = 10^{\log_{10}(5x-5)}$
Simplify:	$x + 4 = 5x - 5$
Add 5:	$x + 9 = 5x$
Subtract x:	$9 = 4x$
Divide by 2:	$x = \frac{9}{4}$
Test the solution of x:	$\log_{10}\left(\frac{9}{4} + 4\right) = \log_{10}\left(\left(5 \cdot \frac{9}{4}\right) - 5\right)$
	$\log_{10}\left(\frac{9}{4} + \frac{16}{4}\right) = \log_{10}\left(\frac{45}{4} - \frac{20}{4}\right)$ ✓

Final solution: $x = \frac{9}{4}$

36) A fossilized leaf contains 15% of its normal amount of carbon 14. How old is the fossil (to the nearest year)? Use 5600 years as the half-life of carbon 14, and $A = A_0 e^{kt}$.

There are two steps to problems like this:

1) Find the value of k based on the half-life of 5600 years.
2) Find how old the fossil is when there is 15% left.

What are the variables?

The formula for exponential decay is: $A = A_0 e^{kt}$, where:

- A is the amount of substance left at time t.
- A_0 is the starting amount of the substance.
- k is the annual rate of decay.
- t is the number of years.

Step 1: Determine the value of k.

We are given: $t = 5600$, $\frac{A}{A_0} = \frac{1}{2}$ (because we are given a "half-life")

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $\frac{1}{2} = e^{5600\,k}$

Take natural logs: $\ln \frac{1}{2} = 5600\,k$

Divide by 5600: $k = \frac{\ln \frac{1}{2}}{5600} = -0.000123776$

Note: For half-life problems, it is always true that:

$$k = \frac{\ln \frac{1}{2}}{half\ life} = \frac{-\ln\ 2}{half\ life}$$

Continued on the next page.

Step 2: Find how old the fossil is when there is 15% left.

We are given: are given: $\frac{A}{A_0} = 15\%$ left, $k = -0.000123776$

Starting equation: $A = A_0 e^{kt}$

Divide by A_0: $\frac{A}{A_0} = e^{kt}$

Substitute in values: $0.15 = e^{(-0.000123776)\cdot t}$

Take natural logs: $\ln(0.15) = -0.000123776 \cdot t$

Divide by (-0.000123776): $t = \frac{\ln(0.15)}{-0.000123776} =$ **15,327 years old**

37) To save for retirement, you decide to deposit \$563 into an IRA at the end of each quarter for the next 35 years. If the interest rate is 5% per year compounded quarterly, find the value of the IRA after 35 years.

The formula taught in class **assumes the deposit is made at the end of the quarter**.

$$S = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{r}{n}}$$

$P =$ the amount of the monthly deposit ($P = \$563$ in this problem)

$r =$ the annual interest rate ($r = .05$ in this problem)

$n =$ the interest compounding period ($n = 4$ for quarterly compounding, 12 for monthly, etc.)

$t =$ the number of years over which the interest accrues ($t = 35$ in this problem)

Then,

$$S = \frac{P\left[\left(1+\frac{r}{n}\right)^{nt} - 1\right]}{\frac{i}{n}} = \frac{563\left[\left(1+\frac{.05}{4}\right)^{4\cdot(35)} - 1\right]}{\frac{.05}{4}} = \$\mathbf{211,351.04}$$

Typically, answers to money problems are expressed to the nearest penny, except when directed otherwise.

Interest:

Interest paid to you (not requested, but may be on the final) is the ending balance minus what you deposited. You deposited \$563 for $(4 \cdot 35) = 140$ quarters.

$$\text{Interest} = \$211{,}351.04 - \$563(140) = \$132{,}531.04$$

38) Solve the system: $\begin{cases} x + y + z = 0 \\ 2x - y + z = -9 \\ x + 2y - z = -4 \end{cases}$

In most cases, you begin solving a set of three equations by selecting pairs of equations and eliminating the same variable in each pair. I'll start this one by eliminating the variable z.

Add 1st and 3rd equations

$$\begin{array}{r} x + y + z = 0 \\ x + 2y - z = -4 \\ \hline 2x + 3y = -4 \end{array}$$

Add 2nd and 3rd equations

$$\begin{array}{r} 2x - y + z = -9 \\ x + 2y - z = -4 \\ \hline 3x + y = -13 \end{array}$$

Continuing:

$2x + 3y = -4$ multiply by (1)

$3x + y = -13$ multiply by (-3)

$$\begin{array}{r} 2x + 3y = -4 \\ -9x - 3y = 39 \\ \hline -7x = 35 \\ \boldsymbol{x = -5} \end{array}$$

Solve for y:

$$3x + y = -13$$
$$3(-5) + y = -13$$
$$-15 + y = -13$$
$$\boldsymbol{y = 2}$$

Solve for z:

$$x + y + z = 0$$
$$(-5) + (2) + z = 0$$
$$-3 + z = 0$$
$$\boldsymbol{z = 3}$$

Finally, test your results in one of the original equations, but not the one used to solve for the third variable solved for above. In this case, we used the 1st equation to solve for our final variable, z. So, we should use either the 2nd or 3rd equation to check our answer.

Second equation:

$$2x - y + z = -9$$
$$2(-5) - (2) + 3 = -9 \ \checkmark$$

Solution: $(\boldsymbol{-5, 2, 3})$

39) Consider two numbers x and y. The sum of their squares is 58, and the difference of the two numbers is 10. Find the two numbers.

$$x^2 + y^2 = 58 \qquad y - x = 10$$

Let's use the Substitution Method

$$y = x + 10 \qquad x^2 + (x+10)^2 = 58$$

$$x^2 + (x^2 + 20x + 100) = 58$$

$$2x^2 + 20x + 42 = 0$$

$$x^2 + 10x + 21 = 0$$

$$(x+7)(x+3) = 0$$

$$x = \{-7, -3\}$$

When $x = -7$, we get:

$y = (-7) + 10 = 3 \;\Rightarrow\; (-7, 3)$ is a solution

When $x = -3$, we get:

$y = (-3) + 10 = 7 \;\Rightarrow\; (-3, 7)$ is a solution

So, the two numbers are: $\mathbf{-7}$ and $\mathbf{3}$ or $\mathbf{-3}$ and $\mathbf{7}$

For 40-42, find the exact value of the trigonometric function.

40) $\cos\frac{10\pi}{3}$

$$\cos\frac{10\pi}{3} = \cos\left(\frac{10\pi}{3} - 2\pi\right) = \cos\left(\frac{4\pi}{3}\right) = -\mathbf{\frac{1}{2}}$$

41) $\sin\frac{17\pi}{2}$

$$\sin\frac{17\pi}{2} = \sin\left(\frac{17\pi}{2} - 8\pi\right) = \sin\left(\frac{\pi}{2}\right) = \mathbf{1}$$

42) $\cot\left(-\frac{\pi}{6}\right)$

$$\cot\left(-\frac{\pi}{6}\right) = \frac{\cos\left(-\frac{\pi}{6}\right)}{\sin\left(-\frac{\pi}{6}\right)} = \frac{\cos\left(\frac{\pi}{6}\right)}{-\sin\left(\frac{\pi}{6}\right)} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}} = -\sqrt{\mathbf{3}}$$

43) Find $\sin t$, if $0 \le t < \frac{\pi}{2}$ and $\cos t = \frac{\sqrt{14}}{4}$. Hint: use the Pythagorean Identity or the Pythagorean Theorem.

$$\sin^2 t + \cos^2 t = 1 \quad \Rightarrow \quad \sin^2 t + \left(\frac{\sqrt{14}}{4}\right)^2 = 1$$

$$\sin^2 t + \frac{14}{16} = 1$$

$$\sin^2 t = \frac{2}{16} \text{ in Q1} \quad \Rightarrow \quad \mathbf{\sin t = \frac{\sqrt{2}}{4}}$$

For #44 – 45: Find a cofunction with the same value as the given expression.

44) $\sin \frac{\pi}{19}$

$$\sin\frac{\pi}{19} = \cos\left(\frac{\pi}{2} - \frac{\pi}{19}\right) = \mathbf{\cos\left(\frac{17\pi}{38}\right)}$$

$\sin\theta = \cos(90° - \theta)$	$\cos\theta = \sin(90° - \theta)$
$\tan\theta = \cot(90° - \theta)$	$\cot\theta = \tan(90° - \theta)$
$\sec\theta = \csc(90° - \theta)$	$\csc\theta = \sec(90° - \theta)$

45) $\csc 52°$

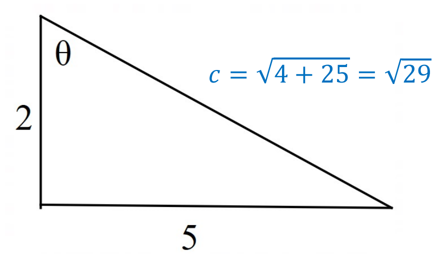
$$\csc 52° = \sec(90° - 52°) = \mathbf{\sec(38°)}$$

46) Find all six trig functions for the angle θ.

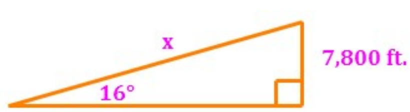
$$\sin\theta = \frac{5}{\sqrt{29}} = \frac{5\sqrt{29}}{29} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{2}{5}$$

$$\cos\theta = \frac{2}{\sqrt{29}} = \frac{2\sqrt{29}}{29} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{29}}{2}$$

$$\tan\theta = \frac{5}{2} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\sqrt{29}}{5}$$



47) A straight trail with a uniform inclination of 16° leads from a lodge at an elevation of 500 feet to a mountain lake at an elevation of 8300 feet. What is the length of the trail, to the nearest foot?



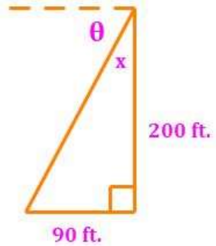
Calculate the height of the triangle as the difference in elevation between the lake and the lodge.

$8{,}300 - 500 = 7{,}800.$

$$\sin 16° = \frac{7{,}800}{x}$$

$$x = \frac{7{,}800}{\sin 16°} = \mathbf{28{,}298} \text{ ft.}$$

48) A building 200 feet tall cast a 90 ft long shadow. If a person looks down from the top of the building, what is the measure of the angle of depression from the top of the building, round to the nearest degree. Assume the person's eyes are level with the top of the building.



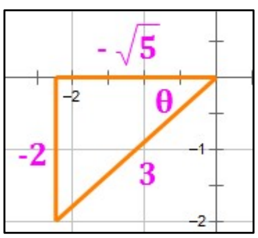
$$\tan x° = \frac{90}{200} = 0.45$$

$$x = \tan^{-1} 0.45 = 24°$$

$$\text{Angle of depression} = \theta = 90° - 24° = \mathbf{66°}$$

For #49 – 50: Find the exact value of the indicated trigonometric function of θ.

49) Find $\sec\theta$ if $\sin\theta = -\frac{2}{3}$, $\tan\theta > 0$

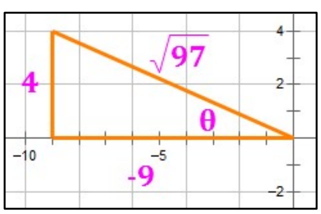


The key on this type of problem is to draw the correct triangle in the correct quadrant. Notice that $\sin\theta < 0$, $\tan\theta > 0$. Therefore θ is in $Q3$.

Notice that the horizontal leg must be: $-\sqrt{3^2 - (-2)^2} = -\sqrt{5}$.

Then, $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{3}{-\sqrt{5}} = -\dfrac{3\sqrt{5}}{5}$

50) Find $\csc\theta$ if $\cot\theta = -\frac{9}{4}$, $\cos\theta < 0$



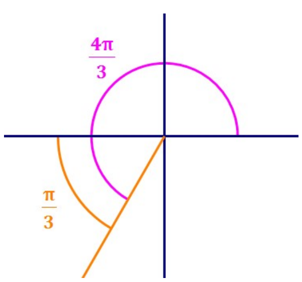
Notice that $\cot\theta < 0$, $\cos\theta < 0$. Therefore θ is in $Q2$.

The hypotenuse has length: $\sqrt{4^2 + (-9)^2} = \sqrt{97}$.

Then, $\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\sqrt{97}}{4}$

For #51 – 53: Use reference angles to find the exact value of the expression. Do not use a calculator.

51) $\sin\frac{4\pi}{3}$



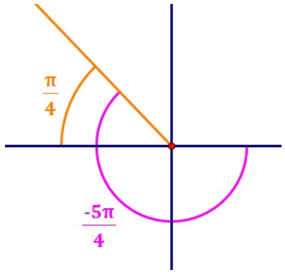
I like to draw the given angle so I can visualize the reference angle and the quadrant it is in.

$\frac{4\pi}{3}$ terminates in Q3. The reference angle is $\frac{4\pi}{3} - \pi = \frac{\pi}{3}$.

The sine function is negative in Q3. So,

$$\sin\left(\frac{4\pi}{3}\right) = -\sin\left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}$$

52) $\sec\frac{-5\pi}{4}$

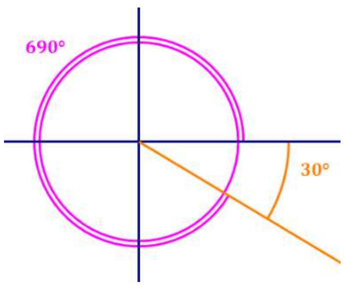


$\frac{-5\pi}{4}$ terminates in Q2. The reference angle is $\frac{\pi}{4}$.

The secant (and cosine) functions are negative in Q2. So,

$$\sec\left(\frac{-5\pi}{4}\right) = -\sec\left(\frac{\pi}{4}\right) = -\frac{1}{\cos\left(\frac{\pi}{4}\right)} = -\sqrt{2}$$

53) $\tan 690°$



The given angle terminates in Q4. The reference angle is $720° - 690° = 30°$.
The tangent function is negative in Q4. So,

$$\tan(690°) = -\tan(30°) = -\frac{\sin(30°)}{\cos(30°)} = -\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

For 54 – 55: Write the partial fraction decomposition of the rational expression.

54) $\frac{9x+8}{(x-6)^2}$

Write the form of the decomposition: $\frac{9x+8}{(x-6)^2} = \frac{A}{x-6} + \frac{B}{(x-6)^2}$

Multiply both sides by $(x-6)^2$: $9x + 8 = A(x-6) + B$

Expand and simplify: $9x + 8 = Ax + (B - 6A)$

Write the simultaneous equations and solve them:

$A = 9$ $\quad$ $B - 6A = 8$

<u>*Solve for B:*</u>

$B - 6(9) = 8$

$B - 54 = 8$

$B = 62$

So, the partial fraction decomposition is:

$$\frac{9x+8}{(x-6)^2} = \frac{9}{x-6} + \frac{62}{(x-6)^2}$$

55) $\frac{32-5x}{x(x-4)^2}$

Write the form of the decomposition: $\frac{32-5x}{x(x-4)^2} = \frac{A}{x} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$

Multiply both sides by $x(x-4)^2$: $32-5x = A(x-4)^2 + Bx(x-4) + Cx$

Expand and simplify: $32-5x = A(x^2-8x+16) + B(x^2-4x) + Cx$

$$-5x+32 = (A+B)x^2 + (-8A-4B+C)x + 16A$$

Write the simultaneous equations and solve them:

$A+B=0 \qquad -8A-4B+C=-5 \qquad 16A=32$

Solve for A:

$16A = 32$

$A = 2$

Then, solve for B:

$A+B=0$

$2+B=0$

$B=-2$

Then, solve for C:

$-8A-4B+C=-5$

$-8(2)-4(-2)+C=-5$

$-16+8+C=-5$

$C=3$

So, the partial fraction decomposition is:

$$\frac{32-5x}{x(x-4)^2} = \frac{\mathbf{2}}{\boldsymbol{x}} + \frac{\mathbf{-2}}{\boldsymbol{x-4}} + \frac{\mathbf{3}}{\boldsymbol{(x-4)^2}}$$

56) Write the **form** of the partial fraction decomposition of the rational expression. $\frac{6x+2}{(x-7)(x^2+x+3)^2}$

$$\frac{6x+2}{(x-7)(x^2+x+3)^2}$$

Note that x^2+x+3 cannot be factored because its discriminant is negative.
That is, $\Delta = b^2-4ac = 1^2-(4\cdot1\cdot3) < 0$.

If x^2+x+3 could be factored, we would need to factor it before setting up the form of the partial fraction decomposition. Watch for this on the test.

Write the form of the decomposition: $\frac{6x+2}{(x-7)(x^2+x+3)^2} = \frac{A}{x-7} + \frac{Bx+C}{(x^2+x+3)} + \frac{Dx+E}{(x^2+x+3)^2}$

I'm glad we don't have to solve this one. It would give a system of 5 equations in 5 unknowns. Yuk!

For 57 – 58: Solve the system by any method.

57) $\begin{cases} x^2 + y^2 = 113 \\ x + y = 15 \end{cases}$

$x^2 + y^2 = 113$ $\quad\quad$ $x + y = 15$

$x = 15 - y$

$(15 - y)^2 + y^2 = 113$

$(y^2 - 30y + 225) + y^2 = 113$

$2y^2 - 30y + 112 = 0$

$y^2 - 15y + 56 = 0$

$(y - 7)(y - 8) = 0$

$y = \{7, 8\}$

When $y = 7$, we get: $x + 7 = 15$, so $x = 8$ $\Rightarrow$ $(8, 7)$ is a solution

When $y = 8$, we get: $x + 8 = 15$, so $x = 7$ $\Rightarrow$ $(7, 8)$ is a solution

So, our solutions are: $\{(\mathbf{7}, \mathbf{8}), (\mathbf{8}, \mathbf{7})\}$

58) $\begin{cases} 3x^2 + 2y^2 = 35 \\ 4x^2 + 3y^2 = 48 \end{cases}$

Let's use the Addition (i.e., Elimination) Method

$3x^2 + 2y^2 = 35$ multiply by (3) $\quad 9x^2 + 6y^2 = 105$

$4x^2 + 3y^2 = 48$ multiply by (-2) $\quad -8x^2 - 6y^2 = -96$

$$x^2 = 9$$

$$x = \{\pm 3\}$$

When $x = -3$, we get:

$4x^2 + 3y^2 = 48$

$4(-3)^2 + 3y^2 = 48$

$36 + 3y^2 = 48$

$3y^2 = 12 \quad \Rightarrow \quad y = \pm 2$

$(-3, -2)$ and $(-3, 2)$ are solutions

When $x = 3$, we get:

$4x^2 + 3y^2 = 48$

$4(3)^2 + 3y^2 = 48$

$36 + 3y^2 = 48$

$3y^2 = 12 \quad \Rightarrow \quad y = \pm 2$

$(3, -2)$ and $(3, 2)$ are solutions

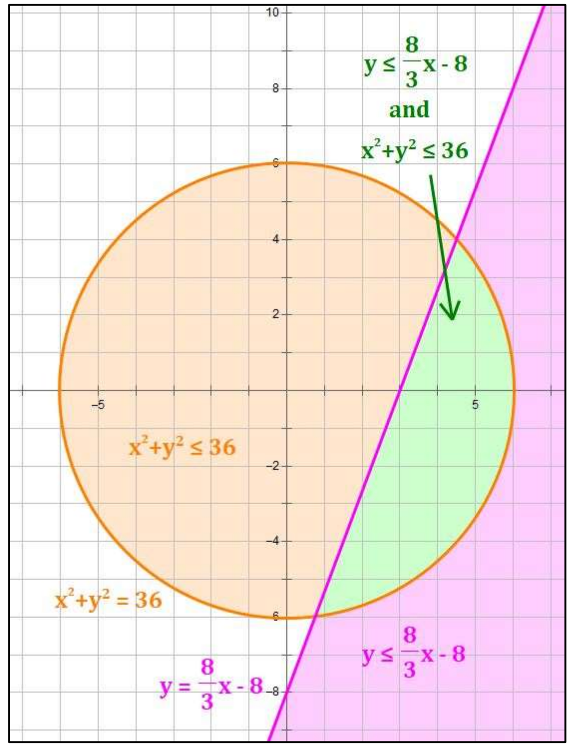
So, the entire solution set is:

$\{(-3, -2), (-3, 2), (3, -2), (3, 2)\}$

For 59 – 60: Graph of the system of inequalities (use the provided graphs for your answers).

59) $\begin{cases} -8x + 3y \le -24 \\ x^2 + y^2 \le 36 \end{cases}$

$x^2 + y^2 \le 36$ (orange and green areas)



- Graph the circle: $x^2 + y^2 = 36$.
- Some points on the curve: $(0,6), (0,-6), (6,0), (-6,0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" sign in the inequality.

$-8x + 3y \le -24$ (violet and green areas)

- Put this in "$y \le mx + b$" form

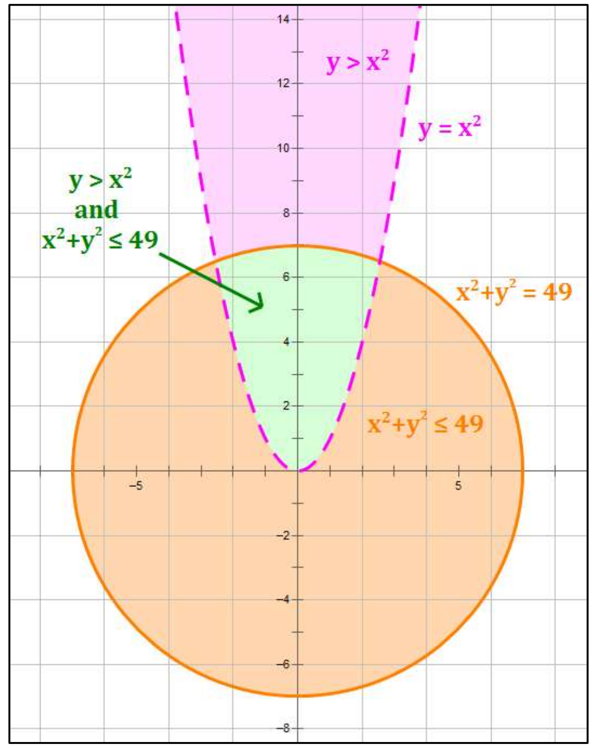
  $3y \le 8x - 24$

  $y \le \frac{8}{3}x - 8$
- Graph the line: $y = \frac{8}{3}x - 8$.
- Some points on the line: $(0,-8), (3,0)$
- The line will be solid because there is an "equal sign" included in the inequality.
- Fill in the portion of the graph below the curve because of the "less than" sign in the inequality.

The green shaded area is the area of intersection of the given inequalities.

60) $\begin{cases} y - x^2 > 0 \\ x^2 + y^2 \le 49 \end{cases}$

$\boldsymbol{x^2 + y^2 \le 49}$ (orange and green areas)



- Graph the circle: $x^2 + y^2 = 49$.
- Some points on the curve: $(0,7), (0,-7), (7,0), (-7,0)$
- The curve will be solid because there is an "equal sign" included in the inequality.
- Fill in the interior of the circle because of the "less than" portion of the inequality.

$\boldsymbol{y - x^2 > 0}$ (violet and green areas)

- Put this in "$y >$" form
  $y - x^2 > 0$
  $y > x^2$
- Graph the parabola: $y = x^2$.
- Some points on the curve: $(0,0), (2,4), (-2,4)$
- The curve will be dashed because there is no "equal sign" included in the inequality.
- Fill in the portion of the graph above the curve because of the "greater than" sign in the inequality.

The green shaded area is the area of intersection of the given inequalities.

61) Graph the constraints and use the objective function to maximize the function.

Objective function: $z = 23x + 8y$
Constraints: $0 \leq x \leq 10$
$0 \leq y \leq 5$
$3x + 2y \geq 6$

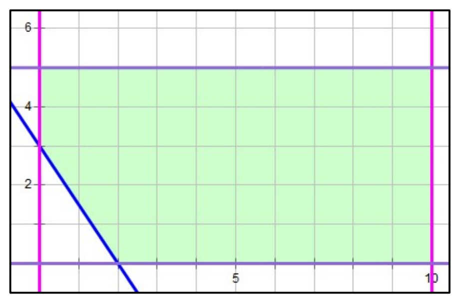
We will need to graph the constraints to find the points of intersection. The maximum and minimum values of the objective function will be at these points.

Constraints: $0 \leq x \leq 10$ $\quad$ $0 \leq y \leq 5$ $\quad$ $3x + 2y \geq 6$

$$y \geq -\frac{3}{2}x + 3$$

Points of intersection (based on the graph): $(2,0), (0,3), (0,5), (10,0), (10,5)$

We are instructed in the statement of the problem to check the Objective Function value (OFV) at each point of intersection, even though **it is obvious that the point $(10, 5)$ will maximize** the OFV because of it's position relative to other points on the graph.



Objective Function values:

Point $(2,0)$: $z = 23x + 8y = 23(2) + 8(0) = 46$

Point $(0,3)$: $z = 23x + 8y = 23(0) + 8(3) = 24$

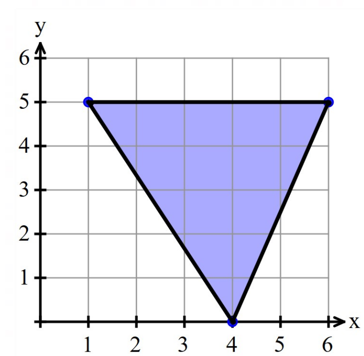
Point $(0,5)$: $z = 23x + 8y = 23(0) + 8(5) = 40$

Point $(10,0)$: $z = 23x + 8y = 23(10) + 8(0) = 230$

Point $(10,5)$: $z = 23x + 8y = 23(10) + 8(5) = 270$

The maximum value of the Objective Function occurs at $(10, 5)$ and is equal to 270.

62) Given the graph shown on the right, which shows the feasible region of a linear program problem. If the objective function is $z = 3x + 4y$, then what is the maximum value?



Objective Function values:

Point $(1,5)$: $z = 3x + 4y = 3(1) + 4(5) = 23$

Point $(4,0)$: $z = 3x + 4y = 3(4) + 4(0) = 12$

Point $(6,5)$: $z = 3x + 4y = 3(6) + 4(5) = 38$

The maximum value of the Objective Function occurs at $(6, 5)$ and is equal to 38.

63) Find a_{20} (the 20th term) for the arithmetic sequence: $11, 4, -3, -10, \ldots$

I like to find $a_0 = a_1 - d$ because $\boldsymbol{a_0}$ **(i.e., the 0th term) is the constant term** in the explicit formula for a_n and $\boldsymbol{d}$ **is the multiplier of $\boldsymbol{n}$**. So, the explicit formula for an arithmetic sequence is always:

$a_n = a_0 + dn$ (note: you need to calculate a_0; it is not given)

For this sequence, $d = 4 - 11 = \mathbf{-7}$, and so $a_0 = a_1 - d = 11 - (-7) = \mathbf{18}$

Then, the explicit formula is: $\boldsymbol{a_n = 18 - 7n}$

Finally, $\boldsymbol{a_{20}} = 18 - 7(20) = 18 - 140 = \mathbf{-122}$

64) Find the sum of the first 20 terms of the arithmetic sequence: -12, -6, 0, 6, ...

Method 1: Think like Gauss

First, we need: $a_{20} = a_1 + 19d = -12 + 19(6) = 102.$ Then:

$$\begin{aligned} S &= -12 \; - 6 \; + 0 \; + 6 + \cdots + 102 \\ S &= 102 + 96 + 90 + 84 + \cdots - 12 \\ \hline 2S &= 90 + 90 + 90 + 90 + \cdots + 90 = 20(90) \end{aligned}$$

Divide both sides by 2, to get

$S = 10(90) = \mathbf{900}$

Method 2: Use the arithmetic series sum formula: $S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n)$

Again, we need $a_{20} = a_1 + 19d = -12 + 19(6) = 102$

$a_1 = -12 \qquad a_{20} = 102 \qquad n = 20$

$S = \left(\frac{n}{2}\right) \cdot (a_1 + a_n) = \left(\frac{20}{2}\right) \cdot (-12 + 102) = 10(90) = \mathbf{900}$

65) Find a_{12} (the 12th term) for the geometric sequence: $a_1 = -5$ and $r = 2$

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$a_1 = -5 \qquad r = 2$

Then, $\mathrm{a_n} = -5 \cdot (2)^{\mathrm{n}-1}$

So, $\mathrm{a_{12}} = -5 \cdot (2)^{12-1} = -5 \cdot 2048 = \mathbf{-10,240}$

66) Write a formula for the general term (the n^{th} term) of the geometric sequence: $3, -\frac{3}{2}, \frac{3}{4}, -\frac{3}{8}, \frac{3}{16}, \ldots$

The general term of a geometric sequence is: $a_n = a_1 \cdot r^{n-1}$

$$a_1 = 3 \qquad r = \frac{-\frac{3}{2}}{3} = \frac{-3}{2 \cdot 3} = -\frac{1}{2}$$

Then, $\boldsymbol{a_n = 3 \cdot \left(-\frac{1}{2}\right)^{n-1}}$

67) Find the sum of the geometric sequence: $\sum_{n=1}^{8}(-4)^n$

Method 1: Add 'em up (note that $r = -4$):

$$-4 + 16 - 64 + 256 - 1{,}024 + 4{,}096 - 16{,}384 + 65{,}536 = \mathbf{52{,}428}$$

Method 2: Use the geometric series sum formula: $S = a_1 \cdot \left(\frac{r^n - 1}{r-1}\right)$

$$a_1 = -4 \qquad r = -4 \qquad n = 8$$

$$S = a_1 \cdot \left(\frac{r^n - 1}{r - 1}\right) = -4\left(\frac{(-4)^8 - 1}{-4 - 1}\right) = \frac{-4 \cdot 65{,}535}{-5} = \mathbf{52{,}428}$$

For 68 – 69: Find the sum of the infinite geometric series, or state that it does not exist:

68) $96 + 24 + 6 + \frac{3}{2} + \cdots$

Method 1: Think like Gauss

$$a_1 = 96 \qquad r = \frac{1}{4}$$

$$\begin{aligned} S &= 96 + 24 + 6 + \frac{3}{2} + \cdots \\ -\frac{1}{4}S &= \quad\ \ -24 - 6 - \frac{3}{2} - \cdots \\ \hline \frac{3}{4}S &= 96 \end{aligned}$$

Note that this series converges because: $|r| = \left|\frac{1}{4}\right| = \frac{1}{4} < 1$.

Multiply both sides by $\frac{4}{3}$, to get

$$S = \frac{4}{3} \cdot 96 = \mathbf{128}$$

Method 2: Use the infinite geometric series sum formula: $S = a_1 \cdot \left(\frac{1}{1-r}\right)$

$$a_1 = 96 \qquad r = \frac{1}{4}$$

$$S = a_1 \cdot \left(\frac{1}{1 - r}\right) = 96\left(\frac{1}{1 - \left(\frac{1}{4}\right)}\right) = \frac{96}{\frac{3}{4}} = \frac{96}{1} \cdot \frac{4}{3} = \mathbf{128}$$

69) $\frac{1}{3} - 1 + 3 - \cdots$

This is a Geometric Series with $r = -3$. Note that:

$$\frac{a_2}{a_1} = \frac{-1}{\frac{1}{3}} = -1 \cdot \frac{3}{1} = -3 \qquad \text{and} \qquad \frac{a_3}{a_2} = \frac{3}{-1} = -3$$

A Geometric Series converges if $|r| < 1$ and diverges otherwise. Therefore, **this series diverges. The sum does not exist.**

70) Use the Binomial Theorem to expand the binomial and express the result in simplified terms: $(2x - 1)^5$

General Formula:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

Step 1: Start with the binomial coefficients

$$\binom{5}{0} + \binom{5}{1} + \binom{5}{2} + \binom{5}{3} + \binom{5}{4} + \binom{5}{5}$$

Step 2: Add in the powers of the first term of the binomial $(2x)$

$$\binom{5}{0}(2x)^5 + \binom{5}{1}(2x)^4 + \binom{5}{2}(2x)^3 + \binom{5}{3}(2x)^2 + \binom{5}{4}(2x)^1 + \binom{5}{5}(2x)^0$$

Step 3: Add in the powers of the second term of the binomial (-1)

$$\binom{5}{0}(2x)^5(-1)^0 + \binom{5}{1}(2x)^4(-1)^1 + \binom{5}{2}(2x)^3(-1)^2 + \binom{5}{3}(2x)^2(-1)^3 + \binom{5}{4}(2x)^1(-1)^4 + \binom{5}{5}(2x)^0(-1)^5$$

Step 4: Simplify:

$$= (1)(32x^5)(1) + (5)(16x^4)(-1) + (10)(8x^3)(1) + (10)(4x^2)(-1) + (5)(2x)(1) + (1)(1)(-1)$$

$$= \mathbf{32x^5 - 80x^4 + 80x^3 - 40x^2 + 10x - 1}$$

71) Find the 8th term in the following binomial expansion: $(x-3y)^{11}$

General Formula:

$$(\boldsymbol{a}+\boldsymbol{b})^{\boldsymbol{n}} = \sum_{\boldsymbol{k}=0}^{\boldsymbol{n}} \binom{\boldsymbol{n}}{\boldsymbol{k}} \boldsymbol{a}^{\boldsymbol{n}-\boldsymbol{k}} \boldsymbol{b}^{\boldsymbol{k}}$$

KEY POINT: Unfortunately, there are several ways to answer this question, based on how the "8th term" is defined. In order to be consistent with the Pearson textbook and homework problems, we must set the value of k to be one less than the number of the term. Using this approach, the first term has $k=0$, so the 8th term has $k=7$. Other sources name the terms differently.

The terms of the binomial expansion of $(a+b)^n$ are typically given by the formula:

$$\binom{n}{k} a^{n-k} b^k$$

Then, using the approach described above for this problem:

$a = x$ $\quad$ $b = -3y$ $\quad$ $n = 11$ $\quad$ $term = 8$ $\quad$ $k = 7$

And, so,

$$\binom{n}{k} a^{n-k} b^k = \binom{11}{7} (x)^{11-7} (-3y)^7 = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} (x)^4 (-3y)^7 = \boldsymbol{-721,710\; x^4 y^7}$$